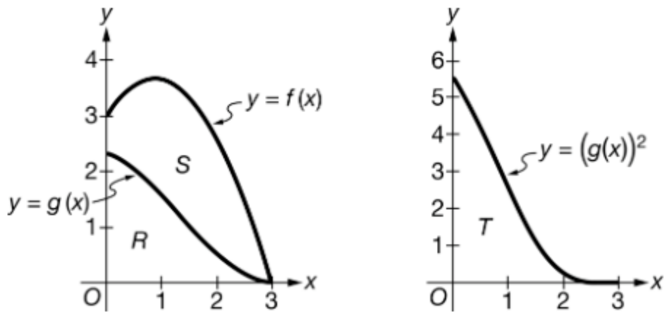
Period 3, April 30, 2025

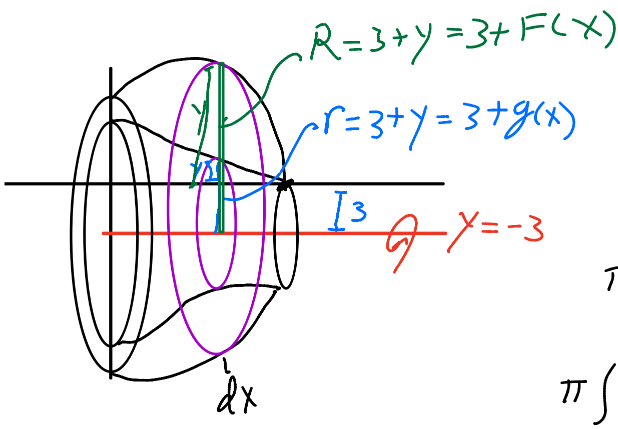


The function f is defined by $f(x) = 3(1+x)^{0.5}\cos\left(\frac{\pi x}{6}\right)$ for $0 \le x \le 3$. The function g is continuous and decreasing for $0 \le x \le 3$ with $g(3) = 0$.

The figure above on the left shows the graphs of f and g and the regions R and S. R is the region bounded by the graph of g and the x- and y-axes. Region R has area 3.24125. S is the region bounded by the y-axis and the graphs of f and g.

The figure above on the right shows the graph of $y = (g(x))^2$ and the region T. T is the region bounded by the graph of $y = (g(x))^2$ and the x- and y-axes. Region T has area 5.32021.

(b) Find the volume of the solid generated when region S is revolved about the horizontal line $y = -3$.



$$\pi\int_0^3 [R^2 - r^2]\,dx$$

$$\pi\int_0^3 [(3+F(x))^2 - (3+g(x))^2]\,dx$$

$$\pi\int_0^3 (3+F(x))^2 - \pi\int (3+g(x))^2\,dx$$

$$\underbrace{\pi\int_0^3 (3+3(1+x)^{0.5}\cos(\tfrac{\pi x}{6}))\,dx}_{\substack{\text{Calculator} \\ 318.8467}} - \pi\int_0^3 (9+6g(x)+g(x)^2)\,dx$$

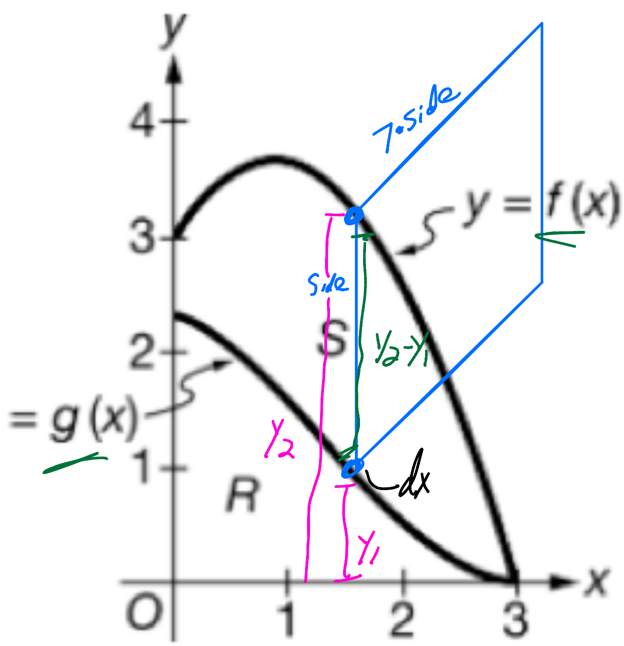
$$-\pi\left[\int_0^3 9\,dx + 6\int_0^3 g(x)\,dx + \int_0^3 g(x)^2\,dx\right]$$

$$-\pi\left[9x\Big|_0^3 + 6\cdot 3.24125 + 5.32021\right]$$

$$318.8467 - \pi[27+6\cdot 3.24125 + 5.32021]$$

cross section dx

(c) Region S is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 7 times the length of its base in region S. Write, but do not evaluate, an integral expression for the volume of this solid.



A = base · height

$A = (F(x)-g(x))\cdot$

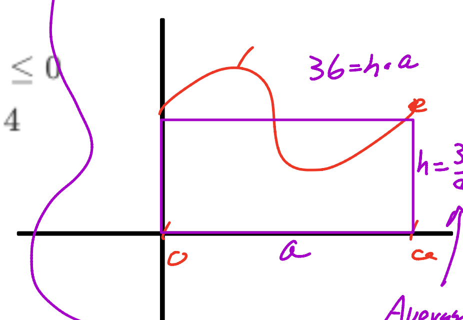
$7(F(x)-g(x))$

$A = 7(F(x)-g(x))^2$

$$\int_0^3 7(F(x)-g(x))^2\,dx$$

$\int F(x)\,dx$ = Area under curve = 36

$F(x)$



$$f(x) = \begin{cases} \sqrt{9-x^2} & \text{for } -3 \le x \le 0 \\ -x + 3\cos\left(\frac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \end{cases}$$

ion defined above.

height of Rectangle — 7 units

(c) Find the average value of f on the interval $-3 \le x \le 4$.

$$\frac{\int_{-3}^{4} F(x)\,dx}{7} = \frac{\int_{-3}^{0} F(x)\,dx + \int_{0}^{4} F(x)\,dx}{7} = \frac{\frac{9\pi}{4}-8}{7}$$

$$\int_{-3}^{0}\sqrt{9-x^2}\,dx + \int_0^4\left(-x+3\cos\frac{\pi x}{2}\right)dx$$

$\frac{\pi r^2}{4}$

$$\frac{9\pi}{4} + \int_0^4\left(-x+3\cos\frac{\pi x}{2}\right)dx$$

$$\frac{9\pi}{4} + \left[-\frac{1}{2}x^2 + \frac{6}{\pi}\sin\frac{\pi x}{2}\right]_0^4 = \frac{9\pi}{4} - 8$$

$\int 3\cos\frac{\pi x}{2}\,dx = \int 3\cos u \cdot \frac{2du}{\pi}$

$u = \frac{\pi x}{2}$

$du = \frac{\pi}{2}dx$

$\frac{2du}{\pi} = dx$

$\frac{6}{\pi}\int \cos u\,du$

$\frac{6}{\pi}\sin u$

$y = \sqrt{9-x^2}$

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$

circle

$r = 3$

need To be contInious

(d) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \le x \le 4$? Justify your answer.

$F'(x) = 0$ or ϕ

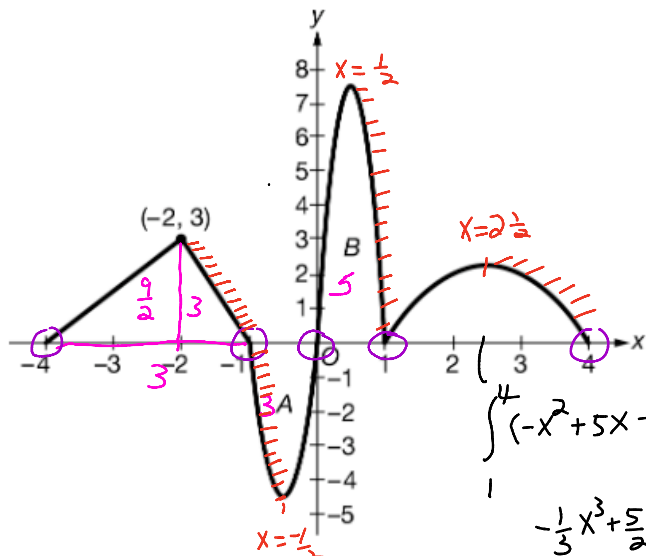
$$F(x) \begin{cases} \sqrt{9-x^2} & -3 \le x \le 0 \\ -x + 3\cos\frac{\pi x}{2} & 0 < x \le 4 \end{cases}$$

$F(0) = \sqrt{9-0^2} = \sqrt{9} = 3$ Continious

$F(0) = -0 + 3\cos\frac{\pi \cdot 0}{2} = -0 + 3 \cdot 1 = 3$

EVT

x	y
-3	
4	
0	
$F'(x) = 0$ or ϕ	



Graph of f

$\int_1^4 (-x^2+5x-4)\,dx$

$-\frac{1}{3}x^3+\frac{5}{2}x^2-4x \Big[_1^4$

$-\frac{64}{3}+40-16-\left[-\frac{1}{3}(1)^3+\frac{5}{2}(1)^2-4(1)\right]$

The continuous function f is defined for $-4 \le x \le 4$. The graph of f, shown above, consists of two line segments and portions of three parabolas. The graph has horizontal tangents at $x=-\frac{1}{2}$, $x=\frac{1}{2}$, and $x=\frac{5}{2}$. It is known that $f(x)=-x^2+5x-4$ for $1 \le x \le 4$. The areas of regions A and B bounded by the graph of f and the x-axis are 3 and 5, respectively. Let g be the function defined by $g(x)=\int_{-4}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(4)$.

$g(0)=\int_{-4}^{0} F(T)\,dT = \frac{9}{2}-3=\frac{9}{2}-\frac{6}{2}=\frac{3}{2}$

$g(4)=\int_{-4}^{4} F(T)\,dT = \int_{-4}^{0} F(T)\,dT + \int_{0}^{4} F(T)\,dT = \frac{3}{2}+5+\int_1^4 (-x^2+5x-4)\,dx$

$\frac{3}{2} + \frac{-64}{3}+40-16+\frac{1}{3}-\frac{5}{2}+4$

$\frac{-63}{3}-\frac{2}{2}+24+4$

$5+ -21-1+24+4=\boxed{11}$

− To + only at X=0

(b) Find the absolute minimum value of g on the closed interval $[-4,4]$. Justify your answer.

=0

X	Y = g(X)
−4	0
4	11
0	3/2

$g'(x)=0 \text{ or } \emptyset$

$g(x)=\int_{-4}^{x} F(T)\,dT$

$g'(x)=F(x)=0$

$g(-4)=\int_{-4}^{-4} F(x)\,dx=0$

(c) Find all intervals on which the graph of g is concave down. Give a reason for your answer.

$g(x)=\int_{-4}^{x} F(T)\,dT$

$g''(x)=$ negative

$g'(x)=F(x)$

$(-2,-\frac{1}{2})\cup(\frac{1}{2},1)\cup(2\frac{1}{2},4)$

$g''(x)=F'(x)=$ Slope of $F(x)$ = Find when slope of $F(x)$ is negative

t (hours)	0	1	2	3	4
$B(t)$ (miles per hour)	1	8	1.5	−5	11

Brandon and Chloe ride their bikes for 4 hours along a flat, straight road. Brandon's velocity, in miles per hour, at time t hours is given by a differentiable function B for $0 \le t \le 4$. Values of $B(t)$ for selected times t are given in the table above. Chloe's velocity, in miles per hour, at time t hours is given by the piecewise function C defined by

$$C(t)=\begin{cases} te^{4-t^2} & \text{for } 0 \le t \le 2 \\ 12-3t-t^2 & \text{for } 2 < t \le 4. \end{cases}$$

$C(0)=0\cdot e^{4-0}=0$

$C(1)=1\cdot e^{4-1^2}=e^3$

(b) At time $t=3$, is Chloe's speed increasing or decreasing? Give a reason for your answer.

$V(T)$ and $a(T)$ Same Sign ($+$ or $-$)

$V(T)$ and $a(T)$ Opposite Signs ($+$ or $-$)

$C'(3)=0-3-2T$

$0-3-2(3)=-9\ m/h^2$

$C(3)=12-3(3)-3^2=12-9-9=-6\ m/h$

both negative Chloe is increasing in speed

(c) Is there a time t, for $0 \le t \le 4$, at which Brandon's acceleration is equal to 2.5 miles per hour per hour? Justify your answer.

MVT

Find a Slope of Points $=2.5$

$m=\frac{11-1}{4-0}=\frac{10}{4}=2.5$

$(0,1)(4,11)$

Function is diff

(d) Is there a time t, for $0 \le t \le 2$, at which Brandon's velocity is equal to Chloe's velocity? Justify your answer.

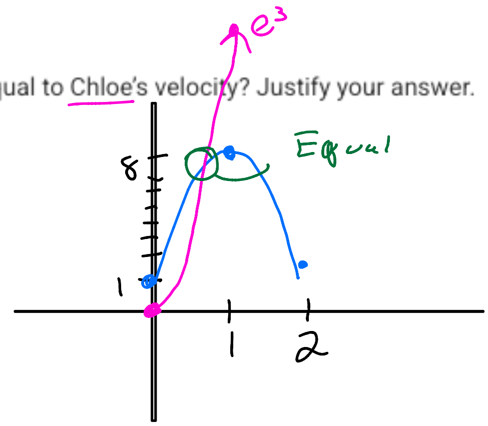
$B(T) = C(T)$

$B(T) - C(T) = 0$

$B(0) - C(0) = 1 - 0 = 1$ positive

$B(1) - C(1) = 8 - e^3 =$ negative

IVT



Consider the curve defined by $2x^2 + 3y^2 - 4xy = 36$.

(a) Show that $\frac{dy}{dx} = \frac{2y-2x}{3y-2x}$.

$x = 6 \quad y = ?$

$2(6)^2 + 3y^2 - 4 \cdot 6 \cdot y = 36$

$72 + 3y^2 - 24y = 36$

$-36 \qquad -36$

$3y^2 - 24y + 36 = 0$

(b) Find the slope of the line tangent to the curve at each point on the curve where $x = 6$.

$x = 6$

$3(y^2 - 8y + 12) = 0$

$3(y-6)(y-2) = 0$

$y = 6$ or $y = 2$

Slope

$(6,2) = \frac{2 \cdot 2 - 2 \cdot 6}{3 \cdot 2 - 2 \cdot 6} = \frac{4-12}{6-12} = \frac{-8}{-6} = \frac{4}{3}$

$(6,6) = \frac{2 \cdot 6 - 2 \cdot 6}{3 \cdot 6 - 2 \cdot 6} = \frac{12-12}{18-12} = \frac{0}{6} = 0$

(c) Find the positive value of x at which the curve has a vertical tangent line. Show the work that leads to your answer.

$\frac{dy}{dx} = \frac{c}{0} = \frac{2y-2x}{3y-2x}$

$\frac{0}{0} \Rightarrow 2y = 2x$

$3y - 2x = 0$

$y = \frac{2}{3}x$

$2x^2 + 3\left(\frac{2}{3}x\right)^2 - 4x \cdot \frac{2}{3}x = 36$

$2x^2 + \frac{4}{3}x^2 - \frac{8}{3}x^2 = 36$

$\frac{6x^2}{3} - \frac{4}{3}x^2 = \frac{2}{3}x^2$

$\frac{3}{2} \cdot \frac{2}{3}x^2 = 36 \cdot \frac{3}{2}$

$x^2 = 54$

$x = \pm\sqrt{54} = +\sqrt{54} = x$

(d) Let x and y be functions of time t that are related by the equation $2x^2 + 3y^2 - 4xy = 36$. At time $t = 1$, the value of x is 2, the value of y is -2, and the value of $\frac{dy}{dt}$ is 4. Find the value of $\frac{dx}{dt}$ at time $t = 1$.

$4x\frac{dx}{dT} + 6y\frac{dy}{dT} - \left[4\frac{dx}{dT} \cdot y + 4x \cdot \frac{dy}{dT}\right] = 0$

$4(2) \cdot \frac{dx}{dT} + 6 \cdot -2 \cdot 4 - \left[4 \cdot -2\frac{dx}{dT} + 4(2) \cdot 4\right] = 8\frac{dx}{dT} - 48 + 8\frac{dx}{dT} - 32 = 0$

$\frac{16\frac{dx}{dT}}{16} = \frac{80}{16}$

$\frac{dx}{dT} = 5$